PROCEEDINGS OF THE MATHEMATICS IN INDUSTRY STUDY GROUP

2024

Mathematics in Industry Study Group South Africa
MISGSA 2024

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The twenty-first Mathematics in Industry Study Group (MISG) in South Africa was held at the Mathematical Sciences Building, University of the Witwatersrand, Johannesburg, from Monday 15 January to Friday 19 January 2024.

The total number of registered participants at the MISG was sixty-five. There were twenty-five Academic Staff, thirty-five Graduate Students and five Industry Representatives. The invited guests were:

Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Western Australia, Australia
Tim Myers	Centre de Recerca Matematica, Barcelona, Spain
Alba Cabrera Codony	University of Girona, Catalonia, Spain

The South African Universities and Institutes which were represented were:

Council for Scientific and Industrial Research
North-West University
Rhodes University
Sol Plaatje University
University of Cape Town
University of Johannesburg
University of KwaZulu-Natal
University of Mpumalanga
University of Pretoria
University of Stellenbosch
University of South Africa
University of the Witwatersrand
University of Venda
University of Zululand

One International University was represented in addition to the Universities of the invited guests,

University of Quebec in Montreal

The MISG was officially opened on Monday morning by the Head of the School of Computer Science and Applied Mathematics at the University of the Witwatersrand, Professor Joel Moitsheki.

The MISG followed the established format for Study Group meetings held throughout the world. South African industry had been approached to submit problems during 2022. Six problems were submitted. On Monday morning each Industry Representative made a twenty-five minute presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked on the problems of their choice. Some participants worked on one problem while others moved between problems and made contributions to several problems. Each problem was co-ordinated by an academic moderator and one or more student moderators. The role of the academic moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short reports at the end of each working day on the progress made that day. The moderators were in contact with the Industry Representatives throughout the meeting. On Friday morning there was a full report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results which were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Graduate Modelling Camp from Wednesday 10 January to Saturday 13 January 2023. The objective of the Graduate Modelling Camp is to provide the graduate students with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working collaboratively in small groups on problems of industrial origin, some of which were presented at previous MISG meetings, at interacting scientifically and at presenting oral reports on their findings. Six problems were presented to the graduate students. The problems and the presenters were:

Problem 1.	Mathematical modelling and optimization for efficient parking allocation
Presenter:	Dr Matthews Sejeso, University of the Witwatersrand, Johannesburg
Problem 2.	Platinum furnace
Presenter:	Professor Neville Fowkes, University of Western Australia, Perth, Australia
Problem 3.	Numerical methods for solving singular integral equations with Cauchy-type kernels
Presenter:	Dr Mathibele Nchabeleng, University of Pretoria
Problem 4.	Beam analysis
Presenter:	Mr Kendall Born, University of the Witwatersrand, Johannesburg
Problem 5.	Mathematical modelling of wind turbines
Presenter:	Professor David P Mason, University of the Witwatersrand, Johannesburg
Problem 6.	Rogue waves
Presenter:	Dr Erick Mubai, University of the Witwatersrand, Johannesburg

The graduate students worked in small study groups on the problem of their choice. Each group presented their results at a report back session on Saturday afternoon.

The sponsors of the Graduate Workshop and the MISG were:

- Hermann Ohlthaver Trust
- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences
- School of Computer Science and Applied Mathematics, University of the Witwatersrand

We thank the sponsors without whose support the Graduate Workshop and the MISG could not have taken place.

STUDY GROUP

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PROBLEM STATEMENTS

Problem 1. The batch pan scheduling problem in a white sugar refinery

Industry: Sugar

Industry Representative: Dr Richard Loubser, Sugar Milling Research Institute, Durban

Problem statement

In a typical white sugar refinery, the sugar is crystalised in batch "Vacuum Pans". These are evaporative crystallisers where evaporation and crystallisation take place simultaneously under vacuum (to be able to operate at a reduced pressure where the boiling temperature is reduced and there is thus less thermal degradation of sucrose). At the end of each batch pan cycle the pan will contain a mixture of crystals and mother liquor with 50% of the sugar (typically) being in crystal form.

The crystallisation is followed by centrifugation to separate the crystal sugar from the mother liquor. There is some water addition at this stage to wash the sugar and dissolve small crystals that can pass into the separated mother liquor (normally called jet). The crystallisation is done in multiple stages with the impurities, predominantly "colour", remaining in the mother liquor.

The colour in the sugar is dependant on the colour remaining in the mother liquor. With 50% of the sucrose being removed from the liquor in each stage of crystallisation the mother liquor colour and sugar colour doubles for each stage. At the same time the quantity of sugar produced in each stage halves. Typical colours would be:

- First sugar: 20
- Second sugar: 40
- Third sugar: 80
- Fourth sugar: 160

These four sugars will be mixed together to give an average final sugar colour of 70.7.

The task is to develop a scheduling program that allocates pans appropriately to be able to process a specified quantity of fine liquor (say 200 tons/hr with a concentration of 76% - thus 76 tons of dissolved sugar) with the following constraints:

- Achieve the smoothest possible total steam demand from all pans.
- Add waiting times between batches to achieve the required scheduling.
- Do not start a batch pan unless there is sufficient liquor/jet in the feed tank to complete the batch.
- There must be sufficient space in the strike receiver to accommodate the contents of a batch pan at the end of its cycle.
- Processing of sugar through the centrifugals must match the requirements for correct proportional mixing of sugars of different grades (so as to maintain the required average sugar colour).
- Determine the minimum number of pans necessary to meet the requirements.

Problem 2. Rogue waves in the Agulhas region

Industry: Shipping

Industry Representative: Mr Morwakoma Matabane, South African Weather Science

Problem statement

Ocean waves in the Sothern African region are caused by the interplay of wind, current, bathymetry, tides. Recently South Africa has been experiencing a high frequency of rogue/freak waves in the Agulhas region (or South Eastern Coastline) that has resulted in loss of life, livelihood, flooding and damage to infrastructure. The problem is centred around finding out the contribution of the wind-current, wind-bathymetry and wind-tide interplay towards making waves extreme/severe in south-eastern parts of Southern Africa.

Problem 3. Modelling a gas chromatograph

Industry: Agriculture, food and beverage, pharmaceutical

Industry Representative: Dr Alba Cabrera-Codony, Laboratory of Chemical and Environmental Engineering. University of Girona, Spain

Problem statement

Gas chromatography (GC) is a widely used technique for separating and analyzing volatile compounds in gas mixtures. GC has a broad range of applications, for example in detecting and quantifying pollutants, pesticides and environmental contaminants in air, water and soil samples. In food and beverage analysis, it can be used to determine the presence and concentrations of flavour compounds or additives. In the pharmaceutical industry, it permits the analysis of drugs, including purity and quantifications of active ingredients. In forensic analysis it plays a crucial role in toxicology and arson investigations. Additionally, in clinical and medical laboratories, GC is used for analyzing blood, urine and other biological samples.

Brief overview of how gas chromatography works:

1. A small amount of the sample containing different compounds is injected into the chromatograph.
2. A carrier gas, usually helium or nitrogen, carries the vaporized sample through the chromatographic column. The column is a long, thin tube: there are two types of column.

    i) Packed Columns, which contain a solid support material coated with a liquid stationary phase and

    ii) Capillary Columns, which are narrower and contain a thin film of stationary phase on the inner wall.

3. Adsorption: This causes separation, based on the different interactions of the compounds, with the stationary phase

4. Separation: As the sample travels through the column, different compounds interact differently with the stationary phase. The extent of interaction depends on factors such as molecular size, polarity and volatility. Compounds that interact strongly with the stationary phase will spend more time in the column, leading to a slower movement through the column. Conversely, compounds that interact weakly will move faster

5. Detection: As each separated compound exits the column, it passes through a detector. The detector responds to the presence of compounds and generates signals proportional to the concentration. Common detectors include flame ionization detectors (FID) electron capture detectors (ECD) and mass spectrometers (MS).

6. The signals from the detector are recorded and analysed to produce a chromatogram, which is a graphical representation of the separation of compounds over time that permits the data analysis

The Study Group is asked to develop a mathematical model of gas chromatography.

Problem 4. Axial strain evaluation without the use of strain gauges

Industry: Mining

Industry Representative: Dr Halil Yilmaz, Rock Mechanics Laboratory, CSIR, Johannesburg

Problem statement

Uniaxial compressive strain testing with the use of axial and lateral strain gauges is one of the most common tests required by the mining industry. Strain gauges are used for the calculation of the Young’s modulus and the Poisson ratio. Young’s modulus is generated from the axial stress versus actual strain data and the Poisson ration is calculated using the lateral strain versus axial strain data. Most universal testing machines are capable of measuring the force and the machine displacement as a crosshead movement in the direction of the applied force. The machines unfortunately cannot measure the deformation of the specimen in the same direction.

Is it possible to infer the axial deformation taking place on the rock specimen without using a strain gauge, by using the data generated by the machine?

A strain gauge can be used only once and is expensive. The materials remain elastic throughout the deformation and linear elasticity described by the generalised Hooke’s law applies.

Problem 5. Tourist attractions capping visitor numbers

Industry: Tourist Sector

Industry Representative: Dr Precious Shabalala, University of South Africa

Problem statement

The continuing tourism growth will eventually result in increased visitation to some destinations/tourist attractions. Certain tourist attractions are limiting the number of visitors they welcome each day. The main reasons are to protect sensitive environments and provide a more enjoyable visitor experience by lessening the crowd. It must be noted that tourism caps have been around for decades and the pandemic travel patterns have encouraged new restrictions to take effect. It is perceived that visitation caps make for an inherently less flexible travel experience but as well as minimizing crowd and putting less strain on staff (which continue to be in short supply), limiting the number of visitors also helps to preserve and conserve natural resources.

The objective is to develop a viable mathematical model to determine the social carrying capacity of a tourist attraction to mitigate the negative impact of over-tourism while providing a high-quality experience for visitors. The model should consider the available infrastructure, activities, natural and cultural resources, and accommodation. The tourist attractions in consideration are Manyeleti Nature Reserve, Mariepskop Nature Reserve, Bushburckrigde Nature Reserve and Injaka Dam.

The solution to the problem will be of global importance to the tourism sector.

The Study Group is asked to develop further a simple model proposed at MISG 2023 to determine the carrying capacity by simulating the visitor flow of a given tourist attraction. The model is derived from traffic flow problems and principles of advection-reaction-diffusion equations. In the case of visitor flow into the tourist attractions, a traffic light network is analogous to a network of different attraction (activities). The model considers the following parameters for a particular tourist attraction: the arrival rate, length of stay, activities and attraction capacity. The aim is to avoid long queues for any activity (attraction) in the network. The Study Group is asked to use simulations to determine the carrying capacity of each tourist attraction, which is the point at which long queues are unavoidable in a network. The model is straight forward, easy to understand, easy to implement and immensely useful. It will require data to simulate real-world situations and identify key model parameters. The Study Group is asked to validate the model by fitting it to real-world situations.

Problem 6. Acoustic signal in wind turbine

Industry: Renewable energy, wind

Industry Representative: Adam Mielke, Technical University of Denmark

Problem statement:

The overall aim is to investigate how to detect faults in wind turbine blades through acoustic emission (AE) measurements. In lab experiments, we move the blade up and down, and detect the sound it makes through analogue sensors. The blade in this experiment is 14.3 m long. Four embedded faults are made during the construction of the blade and AE-sensors are placed around them. The blade is then excited by an arm that moves it in two different directions at once; Flap-wise (bending along the narrow direction of the blade) and edgewise (bending along the wide direction of the blade). The total movement matches an eigenfrequency of 2.305 Hz, which has a quite narrow band and therefore can be regarded as a constant. Note that this is of course a resonance that is avoided in the real world, because the large amount of movement puts a bigger stress on the blade. (If we did not use this frequency in the lab, it would take forever to test the growth of faults.) As the blade is alternating between C-shaped and figure of 8 movements, the edgewise motion has twice the frequency of the flap-wise motion. The conversion from analogue to digital is a bottleneck, so the full signal is not saved. Instead, a number of waveform parameters are saved. The wave packets are strongly dampened, so hardly any acoustic events reach more than the two adjacent sensors. We use a value for the speed of sound along the blade of around 2 km/s, but unfortunately the actual speed of sound is anisotropic (and most likely non-uniform).